Composite Structures 180 (2017) 316–333

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Delamination growth in polymer-matrix fibre composites and the use of fracture mechanics data for material characterisation and life prediction

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ARTICLE INFO

Article history:
Received 27 May 2017
Revised 28 July 2017
Accepted 31 July 2017
Available online 2 August 2017

Keywords:
Delamination growth
Impact damage
Lead delaminations
Fatigue threshold
Modelling
Scatter

ABSTRACT

The growth of delaminations in polymer-matrix fibre composites under cyclic-fatigue loading in operational aircraft structures has always been a very important factor which has the potential to significantly affect the service-life of such structures. The recent introduction by the Federal Aviation Administration (FAA) of a 'slow growth' approach to the certification of composites has further focused attention on the experimental data and the analytical tools needed to assess the growth of delaminations under fatigue loads. Specific attention is given to the test and data-reduction procedures required to determine a 'valid' rate of fatigue crack growth (FCG), da/dN, versus the range of the energy release-rate, ΔG, (or the maximum energy release-rate, G_{max}, in a cycle) relationship (a) to characterise and compare different types of composites, and (b) for designing and lifing in-service composite structures. Now, fibre-bridging may occur behind the tip of the advancing delamination and may cause very significant retardation of the FCG rate. Such retardation effects cannot usually be avoided when using the Mode I double-cantilever beam test to ascertain experimentally the fatigue behaviour of composites, so that a means of estimating a valid (i.e. ideally a 'retardation-free' or, at least, a very low-retardation) relationship is needed. The present paper presents a novel methodology, that is based on a variant of the Hartman-Schijve equation, to ascertain a valid, 'retardation-free', upper-bound FCG rate curves.

1. Introduction

1.1. Background

Polymeric-matrix fibre composites and adhesively-bonded structures are now common in both civilian and military aircraft. As a result, there now is a renewed focus on methods for assessing the in-service performance of damaged composite and bonded structures. For military aircraft these approaches are documented in the United States Joint Services Specification Guidelines JSSG-2006 [1]. The JSSG-2006 document, and the US Composite Materials Handbook CMN-17-3G [2], suggest a building-block approach involving coupon tests, large component tests and finally full-scale fatigue tests.

JSSG-2006 specifically requires that the life of the structure that is subjected to a full-scale fatigue test (FSFT) is equal to, or greater than, twice the design life of the aircraft. Furthermore, Section 4.10 of JSSG-2006 requires that: "Detrimental airframe structural deformations *including delaminations* do not occur at or below 115 percent of design limit load." This means that, for the small initial delaminations that are inherent in the structure, the crack driving force should ideally be beneath the fatigue threshold value. If not, then delamination growth should be slow and such that there is no detectable delamination prior to 115% DLL. Further, any delamination present in the structure must not grow to the point where it causes failure in under two lifetimes.

The US Federal Aviation Administration (FAA) approach to the certification of composite and bonded structures is similar. Prior to 2009 the FAA approach was based on a 'no growth' design philosophy. However, in 2009 the FAA introduced a 'slow growth' approach to certifying composite and adhesively-bonded structures, and also to adhesively-bonded repairs [3].

The JSSG-2006 document also requires a risk of failure assessment to be performed. Indeed, one potential problem with merely certifying via a single FSFT is the large scatter that is often seen in

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http://dx.doi.org/10.1016/j.compstruct.2017.07.097

Nomenclature

a	delamination, i.e. crack, length
a_0	length of the film insert which forms the starter crack in a DCB test
a_s	crack length at which the quasi-static asymptotic value of G is reached
A	constant in the Hartman-Shrive equation
B	constant
da/dN	rate of delamination growth per cycle
D	constant in the Hartman-Shrive equation
DCB	double cantilever beam
DLL	design limit load
FAA	Federal Aviation Administration
FCG	fatigue crack growth
FSFT	full-scale fatigue test
G	energy release-rate
G_{max}	maximum value of the applied energy release-rate in the fatigue cycle
G_{min}	minimum value of the applied energy release-rate in the fatigue cycle
ΔG	range of the applied energy release-rate in the fatigue cycle, as defined below
ΔG	$G_{max} - G_{min}$
$\Delta\sqrt{G}$	range of the applied energy release-rate in the fatigue cycle, as defined below
$\Delta\sqrt{G}$	$\sqrt{G_{max}} - \sqrt{G_{min}}$
$\Delta\sqrt{G_{th}}$	value of $\Delta\sqrt{\Delta G}$ at a value of da/dN of 10^{-10} m/cycle
$\Delta\sqrt{G_{thr}}$	range of the fatigue threshold value of $\Delta\sqrt{G}$, as defined below
$\Delta\sqrt{G_{thr}}$	$\sqrt{G_{thr.max}} - \sqrt{G_{thr.min}}$
$\sqrt{G_{max.thr}}$	threshold value of $\sqrt{G_{max}}$
$\sqrt{G_{min.thr}}$	threshold value of $\sqrt{G_{min}}$
G_{C0}	quasi-static initiation value of G
G_{CSS}	quasi-static steady-state value of G
$G_R(a-a_0)$	functional dependence of the quasi-static energy release rate on $(a-a_0)$
JSSG	Joint Services Specification Guidelines
K	stress-intensity factor
K_{max}	maximum value of the applied stress-intensity factor in the fatigue cycle
K_{min}	minimum value of the applied stress-intensity factor in the fatigue cycle
ΔK	range of the applied stress-intensity factor in the fatigue cycle, as defined below
ΔK	$K_{max} - K_{min}$
ΔK_{thr}	range of the fatigue threshold value of the applied stress-intensity factor, as defined below
ΔK_{thr}	$K_{thr.max} - K_{thr.min}$
N	number of fatigue cycles
n	exponent in the Hartman-Shrive equation
R	displacement ratio $(=\delta_{min}/\delta_{max})$
R^2	the linear correlation coefficient
USAF	United States Airforce
W	strain-energy density
α	constant
β	constant
δ_{max}	maximum displacement applied during the fatigue test
δ_{min}	minimum displacement applied during the fatigue test
ε_{ij}	strain tensor
σ_{ij}	stress tensor
I, II, III	subscripts indicating Mode I (opening-tensile), Mode II (in-plane shear) loads and Mode III (anti-plane shear) loadings

delamination growth [2], even for relatively simple Mode I (opening-tensile) loaded fatigue tests on polymeric-matrix fibre composites [4,5]. For metals the seminal work on the variability in measured fatigue crack growth (FCG) rates is the paper by Virkler et al. [6]. The seminal work on the variability observed in FCG rates, via delamination growth, in composites is [4]. Of equal importance is the finding [7] that this variability can be captured by allowing for small changes in the fatigue threshold term in the Hartman-Schijve variant of the Nasgro equation. This analysis has resulted in a first estimate for the distribution function associated with the delamination thresholds being determined [7]. This, in turn, means that, for any given size delamination, it may be possible to use a variant of the Hartman-Schijve equation to determine the probability distribution associated with the risk of failure.

1.2. Discussion of the above statements

First, a number of questions and comments arise from the above statements:

a) The extensive scatter seen in delamination tests on composites raises the question of whether this implies that no (or limited) observable delamination growth in a FSFT, or in building-block tests that generally do not replicate the actual multi-axial stress state seen in the aircraft, will mean that there will be no (or limited) delaminations seen for in-service aircraft? (It should be noted that the US Composite Materials Handbook [2] states that, whereas a single FSFT to two lifetimes on a metal airframe is sufficient to guarantee safety, for composite airframes a test to fourteen lifetimes would be required to guarantee the same level of assurance.)
b) What happens if an operational aircraft is found to have a delamination that did not arise during the FSFT or in the building-block tests? This could be the case, for example, if peel plies are accidently left in a composite component, if a hole is mis-drilled, or if the operational usage changes sufficiently over time. Indeed, the fact that a mis-drilled hole can result in delamination damage growing during cyclic-fatigue loading was identified in [8]. In such cases the results of the FSFT and the building block tests may not be particularly relevant.
c) What happens if the composite structure is repaired in a manner that was not evaluated in either the FSFT or in the building-block tests?
d) Furthermore, what happens when you have exceeded the life seen in the FSFT divided by the safety factor (which is often two)?

The answer to the first question above is ‘No’, since, for example, failure by delamination has been recorded in an ‘AIRBUS A310’ aircraft [9], which did not arise during either the building block tests or in full-scale fatigue testing. That is to say: the fact that there is no growth in a FSFT, or in the associated building-block tests, does not necessarily mean that delaminations will not arise in service. The answer to the last three questions is obviously that the results of the FSFT and the building-block tests are of limited use if they did not replicate the true multi-axial stress state seen in the aircraft. Indeed, the fact that most 'building-block' tests are subjected to uniaxial loads and hence do not replicate the complex, multi-axial stress states seen in the aircraft structure is clearly a relevant point.

Secondly, it is also commonly thought that if no delaminations/disbonds arise during proof (ultimate) load tests then delamination growth will not arise in-service. This belief is erroneous. Indeed, the importance of working to the correct fatigue threshold, for a given set of stresses and initial flaws, was first highlighted by Schoen et al. [10], who stated:

"During certification of the 'AIRBUS A320' vertical fin, no delamination growth was detected during static loading. The following fatigue loading of the same component had to be interrupted due to large delamination growth. … This demonstrates the importance of using the threshold value instead of the static value for delamination growth in the design of composite structures."

Further details on this delamination test are given in [11]. Consequently, a focus of the present paper is on proposing a means for determining the fatigue threshold values via a fracture-mechanics approach and using the Hartman-Schijve variant of the Nasgro equation.

Thirdly, it is also commonly thought that delaminations will not grow during a FSFT. This belief is also incorrect. The delamination seen in the 'AIRBUS A320' FSFT [11] is one of the first known examples of this phenomenon. Another early example is the delamination seen in the 'F/A-18' fatigue test, see [12] for details. In this instance, a delamination in the composite grew from the last step in the stepped lap-joint, where the epoxy-matrix carbon-fibre composite was adhesively-bonded to the titanium (namely a Ti-6Al-4V alloy) end-fitting, see Fig. 1.

1.3. Analysis of the measured FCG data

The above observations raise some additional questions:

a) How can we determine the fatigue threshold, ΔG_{thr}, values associated with naturally-occurring delaminations?

Here it should be noted that the multiplicity of possible delamination modes means that the fatigue threshold for delamination growth in composites is a failure-locus surface that depends upon the relative proportions of Mode I (opening-tensile), II (in-plane shear) and III (anti-plane shear) loading. This failure locus also appears be a function of the length of the delamination [13,14]. The need to determine the functional dependence of the fatigue threshold ΔG_{thr} on the mode-mix may be of importance in assessing the criterion for 'growth/no growth' of delaminations found in operational aircraft. On other hand, global Mode I loading of composites is invariably the most damaging mode of loading. Hence, using the fracture and fatigue properties of the composite measured under Mode I loading may be considered to give the most conservative estimate of the predicted life of in-service components and structures.

b) How can we determine a valid delamination growth law for subsequent use in predicting residual life and inspection intervals?

As is outlined above, the FAA 'slow growth' approach to aircraft certification requires that delamination growth be both slow and predictable. To be predictable then valid test procedures capable of uniquely characterising delamination growth are needed. In this context it should be noted that, until recently, delamination growth in composites was usually related to the increment in the energy release-rate per cycle, ΔG, or to the maximum value of the energy release-rate, G_{max}, in a cycle. However, in the mid 1990's tests were performed [15] using composite/aluminium laminates employing an R-ratio, R, = 0.1. When the value of da/dN was expressed as a function of G_{max} the different residual stresses that were present in the laminates gave rise to different da/dN versus G_{max} curves, each with different values of the slopes and of the thresholds. This work subsequently revealed, as first proposed by Hartman and Schijve [16] to assess crack growth in metals that, if the term da/dN was expressed as a function of ($G_{max} - G_{thr}$), the various curves collapsed to give a single 'master' relationship.
Indeed, Refs. [17–19] have since revealed that relating da/dN to either ΔG, or G_{max}, is incorrect. Here it should be recalled that, when originally formulating the equations for crack growth, Paris argued that, since Irwin [20,21] had shown that the stress-intensity factor, K, uniquely characterises the near tip stress field, then the rate of FCG should be a function of ΔK and K_{max} [22,23]. In [24] Paris stated:

"Later in 1957 when I first saw the crack tip stress field equations my reaction was immediate that the fluctuation of the crack tip stress intensity factor, K, causing fluctuations of the crack tip stress field surrounding the plastic zone could correlate growth rates."

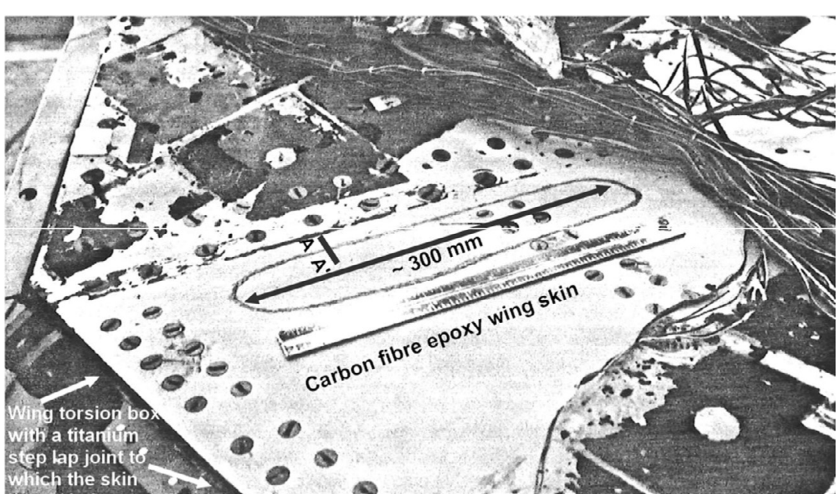

Fig. 1. A large delamination in the composite seen in an early 'F/A-18' fatigue test at 1633 simulated flight hours, see [12] for more details.

Sih, Paris and Irwin [25] subsequently extended the Irwin solution for the crack tip stress field to rectilinearly orthotropic composites. This solution revealed that the near tip stress field for rectilinearly orthotropic composites was uniquely described by $\sqrt{G}$. Thus the logical extension of Paris' FCG law for metals to delamination growth in composites is to express da/dN as a function of $\sqrt{G_{max}}$ or $\Delta\sqrt{G}$ [5,7,12,13,17–19,26–40], and *not* G_{max}, *nor* ΔG, as is commonly done [2,41–55]. (Where $\Delta\sqrt{G}$is given by ($\sqrt{G_{max}}$-$\sqrt{G_{min}}$); and G_{min} is the minimum value of the energy release-rate in a fatigue cycle).

1.4. Alternative fracture mechanics based approaches

Whereas Sih, Paris and Irwin [25] were the first to derive expressions for the energy release-rate associated with a crack in a composite structure, the work of Sih and Chen [56] was the first to show that cracking in the matrix material could be captured via the strain-energy density, W, defined as:

$$W = 1/2(\sigma_{ij}\varepsilon_{ij}) \tag{1}$$

where σ_{ij} and ε_{ij} are the stress and strain tensors, respectively. This approach was subsequently shown to be able to predict delamination failures [57]. The equivalence of using energy release-rate approaches and energy density approaches to predict the failure of impact damaged composites was first shown in [58]. The use of the strain-energy density for the fatigue design of composite structures was pioneered by McDonnell Douglas [59,60]. However, in recent years Lazzarin and co-workers [61,62] have established that brittle fracture appears to be governed by $\Delta\sqrt{W}$, rather than by ΔW. This finding mirrors the above discussion that delamination growth is governed by $\Delta\sqrt{G}$, rather than by ΔG.

To further illustrate how expressing da/dN as a function of ΔG (and ΔW) conflicts with the original Paris formulation, consider the long and small crack-growth results for da/dN versus ΔK data presented in [63] for FCG in 7050-T7451 aluminium alloy, see Fig. 2. The R-ratio, R, = 0.33 data shown in Fig. 2 is from [64] whilst the NASA R = 0.4 data is from the Nasgro data base. The initial crack lengths associated with the small crack data lie between approximately 3 μm to 20 μm and the final crack lengths are of the order of 10 mm. Here we see that, as noted in [63,65,66], the growth of small cracks essentially conforms to a Paris-like equation and is largely R-ratio independent. Noting that for metals and orthotropic materials [25] that G is proportional to K^2, we see that plotting the data given in Fig. 2 as a function of $\Delta(K^2)$ transforms Fig. 2 from a clear and well-ordered figure to a near chaotic plot, see Fig. 3. It also makes it appear that for a given value of the crack driving force, which in this instance is $\Delta(K^2)$, tests performed at R = 0.1 are more severe than tests at R = 0.7 even though the later will have a greater K_{max} value. This phenomenon, i.e. that when plotting da/dN as a function of $\Delta(K^2)$ tests at low R ratios appear to be more severe than tests at high R ratios, also follows from the crack growth data presented in [67] and, as previously remarked is also seen from plotting delamination growth as a function of ΔG, see [17,18,43]. Therefore, it is best to follow the natural extension of the original Paris hypothesis [22,24] and relate da/dN to $\Delta\sqrt{G}$ (or $\sqrt{G_{max}}$), rather than to ΔG or G_{max} or alternatively, as first suggested in [61,62] to $\Delta\sqrt{W}$ rather than by ΔW.

It should be also be noted that other fracture-mechanics approaches to this problem of uniquely characterising delamination growth may be pursued. For example, a potentially interesting alternative approach is the use of a fracture-mechanics to model the extent of fibre-bridging [68–70], which is a main cause of FCG retardation, i.e. the slowing down of the crack growth rate under cyclic-fatigue loading due to some intervening mechanism.

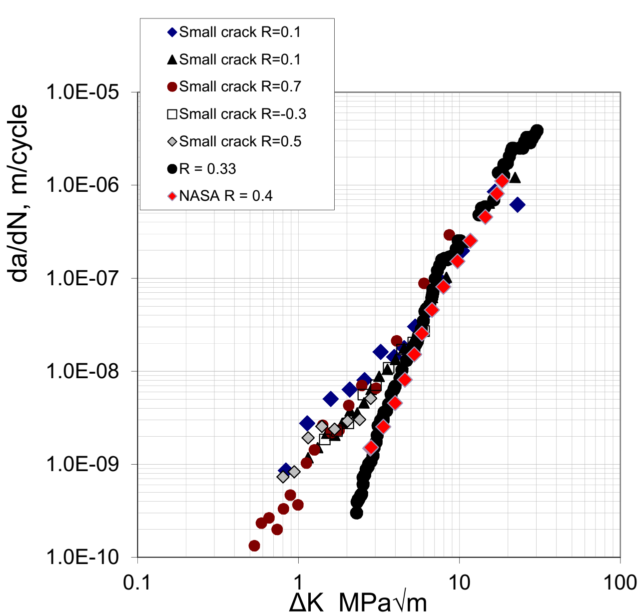

Fig. 2. Comparison of long and small crack growth data for a range of R-ratios for FCG in 7050-T7451 aluminium alloy, from [63,66].

1.5. Scatter in the measured FCG rate data

Whereas the paper by Virkler [6] is widely acknowledged as the seminal study on the variability (i.e. scatter) in FCG rates in metals, for composites that honour goes to Murri [4]. In [4] Murri presented da/dN versus G_{max} curves for delamination growth in thirty-nine Mode I double-cantilever beam (DCB) test specimens which were prepared using IM7/8552 carbon-fibre reinforced-plastic (CFRP). These IM7/8552 specimens were fabricated from composite material obtained from two different sources. The results of the fatigue tests using the CFRP material from each source were used to investigate the variability in the delamination growth rates. Murri [4] subsequently presented a similar study for delamination growth in IM7/977-3, which is another type of CFRP [41]. In all cases, Murri recorded significant scatter in the measured test data. The large scatter that Murri measured, which is typically associated with fatigue delamination growth in composites, has also been noted, for example, in CMH-17-3G [2]. It was subsequently shown [7] that, for tests on both IM7/977-3 and IM7/8522, the scatter in the da/dN versus G_{max} curves presented in [4,41] could be captured by the Hartman-Schijve variant of the Nasgro equation by merely allowing for relatively small changes in the threshold term, G_{thr}. That is to say, that for each material the various curves now all fell onto a single, linear, 'master' relationship when using the Hartman-Schijve variant to plot the measured FCG rates versus a function of the energy release-rate, as described in detail below.

1.6. Aims of the present paper

In this paper we will first discuss the nature of the delamination growth histories that are associated with the growth of delaminations in composites under cyclic-fatigue loadings that initiate and grow from holes and ply drop-offs, as well as the nature of the growth histories associated with impact damage. It is shown that as the delaminations grow they can experience significant retardation. However, the lead damage growth history, which [71] defines as the growth history associated with the fastest growing damage, shows little apparent retardation. Further, as first shown in [71] for

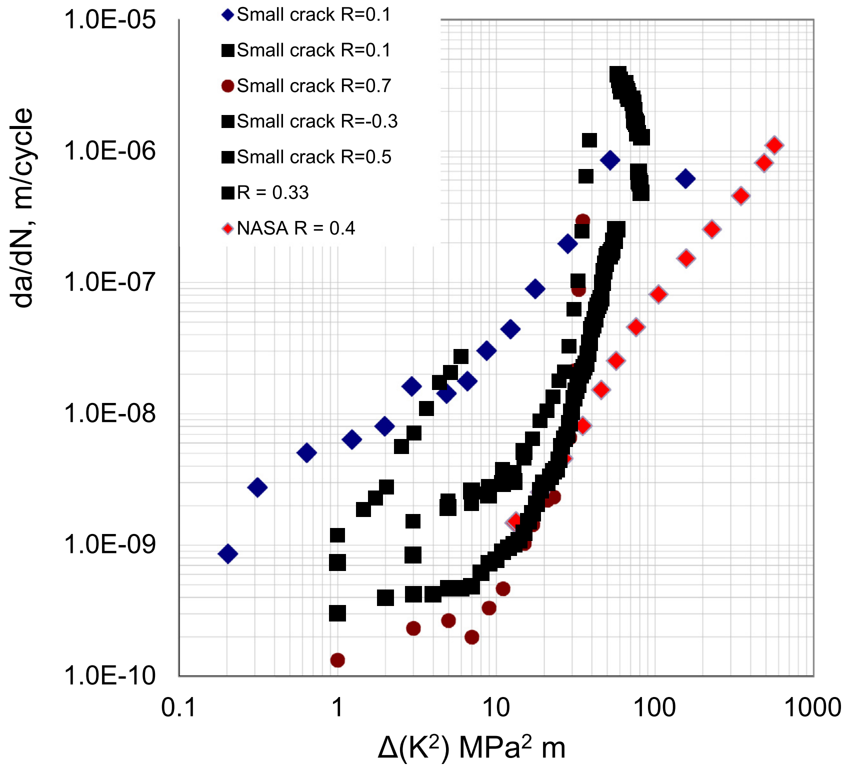

Fig. 3. Comparison of long and small FCG data replotted from Fig. 2 as a function of $\Delta(K^2)$ for 7050-T7451 aluminium alloy.

the growth of impact damage, the lead damage growth history is often nearly exponential in character. It will be shown that these observations for the FCG rate histories associated with the fastest growing (i.e. lead) damage in these composites are consistent with the USAF approach to assessing the probability of failure in metallic airframes [72,73].

Secondly, the effect of retardation on Mode I DCB tests performed under cyclic-fatigue loading in order to determine the relationship between the FCG rate, da/dN, versus a function of the strain-energy release-rate, G, is considered, where a is the crack length and N the number of fatigue cycles. Such retardation can be attributed to the fibre-bridging that typically occurs behind the advancing crack tip in the DCB test.

Thirdly, it is proposed that, to be consistent with the growth of lead delaminations as observed in aircraft structures, and hence to be considered 'valid', the test data from the building-block tests employed, such as the DCB test, should exhibit no, or only very minimal, retardation. If this is not the case, then such fatigue data cannot reliably be used for (a) the characterisation and comparison of composite materials, and (b) designing and lifing in-service composite aircraft structures where 'material allowable' properties have to be inputted into a delamination growth analysis.

Fourthly, it will be shown that valid test data cannot, with any certainty, presently be obtained directly from the measurements undertaken during the fatigue test, due to the fibre-bridging that typically occurs behind the advancing crack (i.e. delamination) tip in the DCB test.

Finally, however, it will be shown that such test data, where no, or only very minimal, retardation is present, can be determined by using a variant of the Hartman-Schijve approach; and that the proposed methodology may also take into account the typical scatter that arises during the course of the experimental work. Thus, this methodology enables the valid, upper-bound FCG rate curves to be ascertained.

2. Damage in composites and the subsequent fatigue growth of lead delaminations

Let us first address the nature of the growth of delamination damage under subsequent fatigue loadings that may arise as a result of initial 'damage' arising from (a) a hole being mis-drilled, (b) a ply drop-off, and (c) impact damage.

2.1. Mis-drilled holes

The early (1979) USAF study [8] pioneered the understanding of delamination growth at a fastener hole. This study presented delamination growth data from a simulated mis-drilled hole. Namely, they employed a 9.5 mm diameter fastener hole in a 76 mm wide panel for a 24 ply $(0_2/+45/0_2/-45/0_2/45/0_2/-45/0)_s$ T300/5208 CFRP composite laminate tested under constant amplitude fatigue loading with a maximum stress of 241 MPa and an R-ratio, R, = −1. They also studied delamination growth data, from a 9.5 mm diameter fastener hole in a 76 mm wide, 32 ply $(0/+45/90/-45_2/90/+45/0)_{2S}$ quasi-isotropic T300/5208 CFRP composite laminate tested under constant amplitude fatigue loads with a maximum stress of 152 MPa and R = -1. The resultant delamination growth histories for these two different CFRP laminate lay-ups are shown in Figs. 4 and 5. This study was one of the first in which the extensive scatter associated with delamination growth under fatigue loading was evident. It was also one of the first papers where the retardation effects, that can arise and influence the growth of large delaminations that may develop from initial damage in the composite, could be readily appreciated, see Figs. 4 and 5.

Figs. 4 and 5 reveal that the fastest growing delamination, i.e. the lead delamination, grows in a near exponential fashion and shows little, if any, retardation effect. That is to say that the delamination length versus cycles curve associated with the fastest growing delamination is exponential with little, if any, reduction

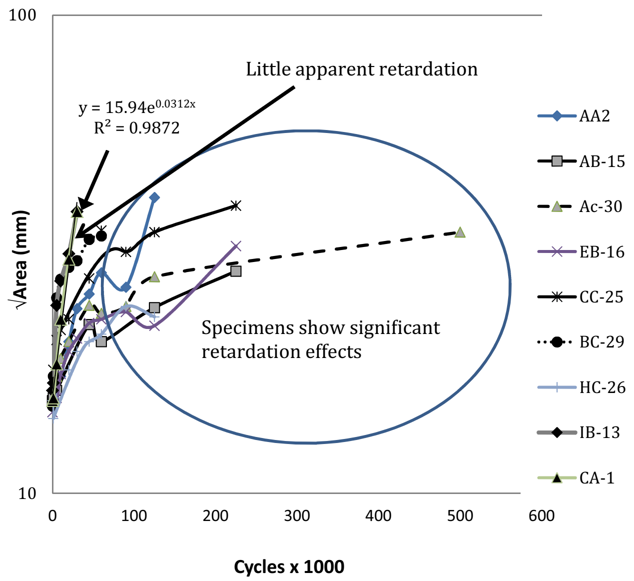

Fig. 4. Delamination fatigue-growth histories from a mis-drilled hole for a number of replicate 24 ply $(0_2/+45/0_2/-45/0_2/45/0_2/-45/0)_s$ T300/5208 CFRP composite laminates tested at a maximum fatigue stress of 241 MPa.

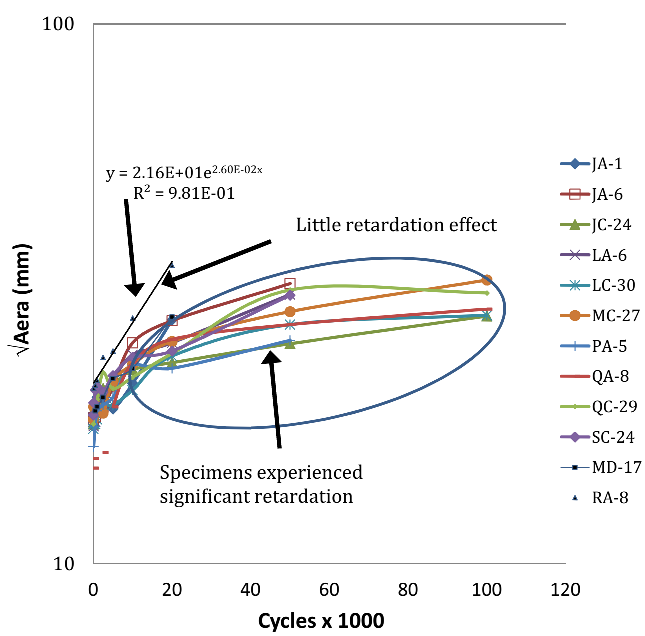

Fig. 5. Delamination fatigue-growth histories from a mis-drilled hole for a number of replicate 32 ply quasi-isotropic T300/5208 CFRP composite laminates tested at a maximum fatigue stress of 152 MPa.

in slope as the number of cycles increases and the fatigue crack grows, see Figs. 4 and 5. This observation is important since the life of a composite airframe is determined by the fastest possible growing delamination, which as can be seen experiences little apparent retardation. The observation of near-exponential growth histories associated with the fastest growing (lead) damage state means that the USAF approach to assessing the probability of failure in metallic airframes [8] may also be applicable to delamination growth in composite airframes.

2.2. Ply drop-off

The nature of the delamination growth history associated with delaminations that arise and grow, under constant amplitude tests at R = −1, from a ply drop-off is shown in [74] for CFRP composite laminates fabricated from unidirectional plies and from bi-directional weaves. Tests were performed at maximum fatigue loads of 22.2, 33.4 and 44.5 kN and the resultant delamination growth histories are shown in Figs. 6 and 7. Here we again see that the growth of the fastest delaminations are essentially exponential and that there is little evidence of retardation, i.e. the length versus number of cycles curves show little reduction in their slope as the delaminations grow, see Figs. 6 and 7.

2.3. Impact damage

As noted above, the experimental data presented in [8] was the first to reveal that, whereas extensive retardation can occur for delaminations that grow from a hole, the fastest growing (i.e. the lead) delamination grows exponentially with little apparent retardation, i.e. little reduction is recorded in the slope of the length versus number of cycles as the delamination grows. Further, this study was also the first to show that this phenomena also held for the fatigue growth of delaminations from induced, initial, impact damage. This phenomenon, i.e. that the growth of the ‘lead fatigue crack resulting from the impact damage’ is essentially exponential and therefore experiences little, if any, retardation, has also been reported for the delamination growth histories presented in [75] for 56 ply XAS-914C CFRP composite laminates which have suffered impact damage and then have been fatigue tested under a FALSTAFF flight-load spectrum with a maximum fatigue stress of 256 MPa, see Fig. 8. (FALSTAFF is an industry standard fighter-aircraft flight-load spectrum). Here it should be noted that, as first shown in [71], growth of the lead damage in the composites was again exponential.

These various examples reveal that the growth of delamination damage under fatigue loads from initial impact induced-damage may experience severe retardation effects. Nevertheless, it is the fastest growing (i.e. lead) damage which sets the operational life, inspection intervals and the repair schedule. In this context we have seen that the fastest growing damage, i.e. the lead damage, experiences a near-exponential growth history with little, if any, apparent retardation.

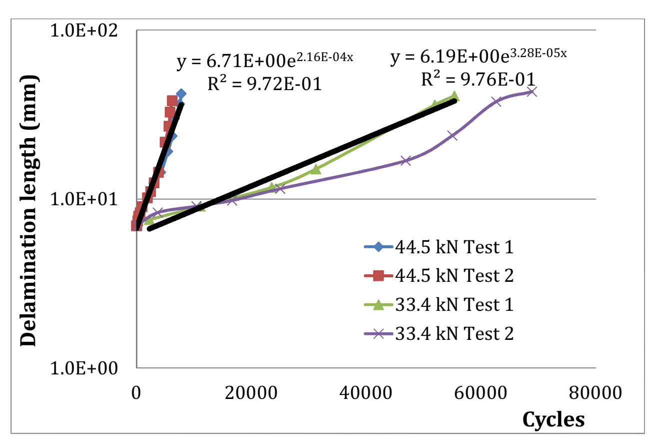

Fig. 6. Delamination fatigue-growth histories from a ply drop-off for unidirectional CFRP composite laminates tested at maximum fatigue loads of 44.5 or 33.4 kN.

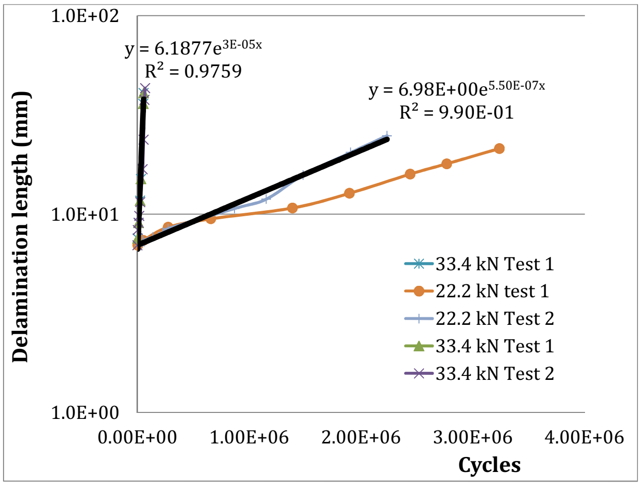

Fig. 7. Delamination fatigue-growth histories from a ply drop-off for woven CFRP composite laminates tested at maximum fatigue loads of 33.4 or 22.2 kN.

3. Delamination fatigue growth curves

3.1. Introduction

To be able to theoretically predict the effect of the FCG rate on the fatigue life of aircraft structures we first need to establish an appropriate delamination growth law based upon the concepts of fracture mechanics. The use of Mode I DCB tests is the most common approach employed to characterise delamination growth as a function of the energy release-rate, G [2,4,5,19,26,38–55,76–78]. However, a major problem with this method is the large retardation that can develop due to fibres bridging the delamination [4,5,13,14,36,42,49]. (A detailed review of the effects of fibre-bridging and the current test standards is given in [78].)

Indeed, when discussing fibre-bridging the ISO test standard ISO 15024 [76] for the DCB test specimen subjected to quasi-static loading states:

> *"This fibre bridging is an artefact of the DCB test and is not representative of the composite material tested."*

This statement is similar to that contained in the ASTM test standard ASTM D 6115–97 [65] for fatigue delamination-onset testing using the DCB test specimen, viz:

> *"Fibre bridging inhibits the fatigue delamination growth resulting in slower growth rates than if there was no bridging. This results in artificially high threshold values where the delamination ceases to grow or grows very slowly."*

These statements reflect the generally held view that fibre-bridging is an artefact of the DCB test and, as we have seen in Section 2, does not reflect how lead delaminations, i.e. delaminations that determine the life of a composite airframe structure, behave. Thus, from an operational standpoint the challenge is to develop tools that can be used to determine, or estimate, the 'retardation-free' fatigue behaviour. By this we mean the da/dN versus $\Delta\sqrt{G}$, or $\sqrt{G_{max}}$, curve which exhibits the fastest possible growth rate from DCB tests when no retardation of the FCG rate, due to fibre-bridging, is present. Obviously, any characterisation studies and comparisons of the fatigue behaviour of composite materials should also be based on the 'retardation-free' fatigue behaviour.

3.2. DCB quasi-static tests

The apparent increase in fracture toughness due to fibre-bridging, measured using the Mode I DCB test subjected to quasi-static loading, has led several authors [4,13,14,42] to express the apparent toughness, G_{CR}, as a function the length, a, of the advancing delamination by the amount by which the delamination exceeds its initial length, a_0. The proposed expressions are of the form shown below, and it should be noted that these present discussions are all concerned with Mode I failure, and hence, the subscript 'I' has been omitted for clarity:

$$\begin{aligned} G_{CR} &= G_{C0} + B(a - a_0) \quad &&\text{for } a < a_s \\ &= G_{RSS} &&\text{for } a > a_s \end{aligned} \tag{2}$$

where a_0 is the length of the relatively thin film inserted in the mid-plane of the DCB test which is used to simulate an initial delamination, B is a constant, G_{C0} is the corresponding 'initiation value' at which the onset of crack growth is recorded, G_{RSS} is the asymptotic, i.e. steady-state, value of G_{CR} and a_s is the delamination length at which this asymptotic value is achieved. (This starter film-insert

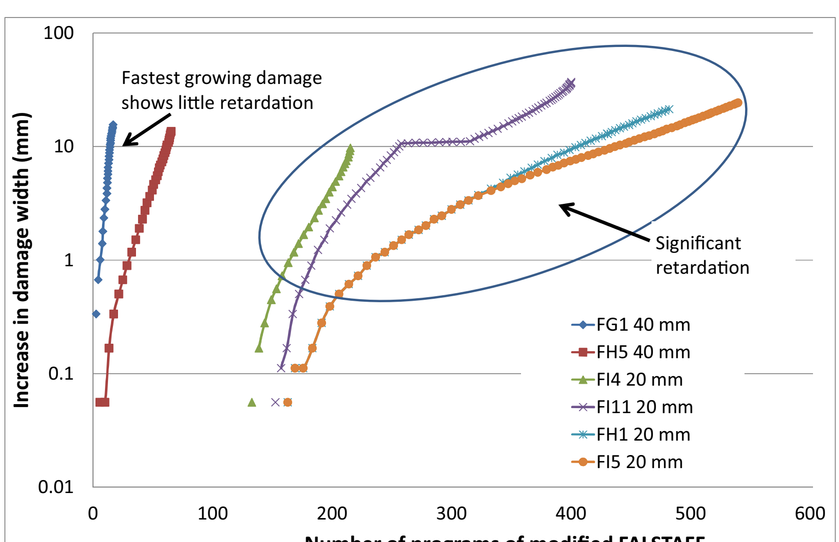

Fig. 8. Delamination fatigue growth histories from initial impact damage for 56 ply XAS- 914 C CFRP composite laminates. The composite laminates had been subjected to impact damage of 20 and 40 mm in diameter before being tested at a maximum fatigue stress of 256 MPa, . adopted from [75]

of length a_0 is typically a very thin film of a fluoropolymer.) Notwithstanding, several difficulties arise in conducting and analysing the results from quasi-static DCB tests and these difficulties are very relevant to using the DCB test to obtain reliable cyclic-fatigue data.

First, the initial, starting delamination of length, a_0, is typically always grown by a small increment to give a relatively sharp 'pre-crack' before the actual test is started. This is to avoid starting the actual DCB test directly from the relatively blunt insert-film, which also typically has a polymer-matrix rich region ahead it. These two factors may well give rise to a false over-estimate of the value of G_{IC0}. Further, the apparent value of G_{IC0} can also be a function of the length of the pre-crack that is generated prior to the start of the actual test, i.e. the length, a-a_0, to which the delamination is grown from the insert-film prior to undertaking the actual test. This has been shown to occur in [40,48,49,52]. This effect arises due to fibre-bridging developing across the delamination as the crack advances during the pre-cracking procedure. Thus, when the actual DCB test is started, crack growth is retarded and the value of the initiation value, G_{C0}, is again over-estimated. Thus, in trying to determine the true value of the G_{C0}, there are two challenges: (a) the need to grow a pre-crack to avoid overestimating the value of G_{C0} due to a relatively blunt crack tip being present immediately ahead of the film insert but (b) to avoid growing the pre-crack too long so that significant fibre-bridging develops across the delamination before the test is actually started. Both of these effects typically lead to an overestimate of the value of G_{C0}. In this context, it should be noted that the procedure outlined in the ISO standard [76] for quasi-static loading tests states that the pre-crack should be grown to a length of $(a - a_0)$ of no more than 3–5 mm from the insert, and that it is practically very difficult to grow the pre-crack to a defined length significantly shorter than prescribed in this standard.

Secondly, the initiation value, G_{C0}, tends to a lower-bound and constant value as the length of the 'pre-crack' tends to zero. However, the results in the literature [4,52] do suggest that the steady-state value of G_{RSS} and the steepness of the R-curve may be a function of the 'pre-crack' length that is employed, since relatively long pre-cracks tend to be associated with extensive fibre-bridging which has already developed across the faces of the delamination before the DCB test is actually started. Indeed, it appears that, dependent upon the type of composite material, there can be variations in G_{RSS} of up to approximately 35% as a function of the length of the 'pre-crack' that was employed before the actual start of the test.

Finally, it should be noted that the selected thickness of the arms of the DCB test specimen may also influence the outcome of the fracture toughness tests and hence may influence delamination growth [79]. Although the value of the initiation value, G_{C0}, is found to be independent of the thickness of the arms of the specimens, both the plateau fracture energy, G_{RSS}, and the steepness of the R-curve may be increase as the thickness of the arms of a unidirectional lay-up CFRP DCB test is increased. However, the opposite effect was found in the same type of composite material but with a multi-directional lay-up. These effects were ascribed to fibre-bridging, which may increase, or decrease, in extent as the thickness of the arms of the DCB test is increased, dependent upon the type of fibre lay-up present in the composite laminate.

3.3. DCB cyclic-fatigue tests

3.3.1. Introduction

Since there is currently no recognised standard for measuring the FCG rate from Mode I DCB tests subjected to fatigue loading, then this has led to several variants of the experimental testing procedure having been proposed and evaluated. Further, it has been shown that there is considerable scatter in the FCG rate curves obtained from DCB tests subjected to fatigue loading. These aspects are discussed below and then some conclusions are drawn from these discussions.

3.3.2. Effect of pre-crack length

First, one difference of test methodology that has been investigated is the method of creating the 'sharp' pre-crack from the initial film-insert. To this end the use of initial quasi-static loading or via fatigue loading has been studied. The experimental data presented in [13,14] revealed that these two different approaches can yield different delamination FCG rate curves. The conclusion reached in this study was:

> *"In these tests, bridging caused by quasi-static delamination is more obvious than in fatigue delamination."*

However, in each case there was only one specimen tested. Thus, given the large scatter that can occur in DCB tests [4,5,31,32], see Fig. 9, this finding must be considered as inconclusive.

Secondly, in the case of employing the DCB test to study FCG rates, the dependence of the level of retardation that arises from the selected length of the initial pre-crack, i.e. $(a - a_0)$, is aptly illustrated in Fig. 10. This figure presents the da/dN versus $\Delta\sqrt{G}$ curves given in [13] for DCB tests where prior to the actual fatigue test the pre-crack delamination was grown from the initial film-insert a distance of (a-a_0) of values of 4.1, 12.7 and 51.3 mm, i.e. before measurements associated with the subsequent fatigue test were taken. (It should be noted that, as commented above, the procedure outlined in the ISO standard [76] for quasi-static loading tests states that the pre-crack should be grown to a length of $(a - a_0)$ of no more than 3–5 mm from the insert, and that it is practically very difficult to grow the pre-crack to a defined length significantly shorter than prescribed in this standard.) The results in Fig. 10 clearly demonstrate that the level of retardation is a strong function of $(a - a_0)$, i.e. the da/dN versus $\Delta\sqrt{G}$ curve shifts to the right as the value of $(a - a_0)$ is increased. This reflects the fact that longer pre-cracks give rise to greater retardation of the FCG rate. This was considered [14] to be due to more extensive fibre-bridging developing across the pre-crack as its length was increased. In turn, this higher degree of fibre-bridging gave rise to a greater retardation of the FCG rate as the value of $(a - a_0)$ was increased, as may be seen in Fig. 10.

Thirdly, a procedure [13] that has been proposed to overcome this retardation effect is to 'cut' the fibres that are bridging the pre-crack before starting to record the FCG data, with the idea of now having no, or only very limited, fibre-bridging hopefully present across the starting delamination. This approach was partially successful in that, as shown in Fig. 10, this cutting of the bridging-fibres has moved the da/dN versus $\Delta\sqrt{G}$curve to the left. However, the resultant 'cut fibres' curve still exhibits greater retardation than was seen for tests using specimens with the smallest initial pre-cracks [13], see Fig. 10. This may well have been because it was not possible to cut all of the fibres bridging the pre-crack prior to the DCB fatigue being conducted.

3.3.3. Effect of the starting value of G_{max}

Hojo et. al. [47,48] were the first to note that, for a given R-ratio, the da/dN versus G_{max} curve determined from DCB tests subjected to fatigue loading was not unique but depended on the starting value of G_{max}. This dependency was subsequently confirmed in [4,42]. As a result it has been argued [47,48] that for real aircraft structures (from a design and from a sustainment/maintenance perspective) it is important to use the da/dN versus G_{max} curve

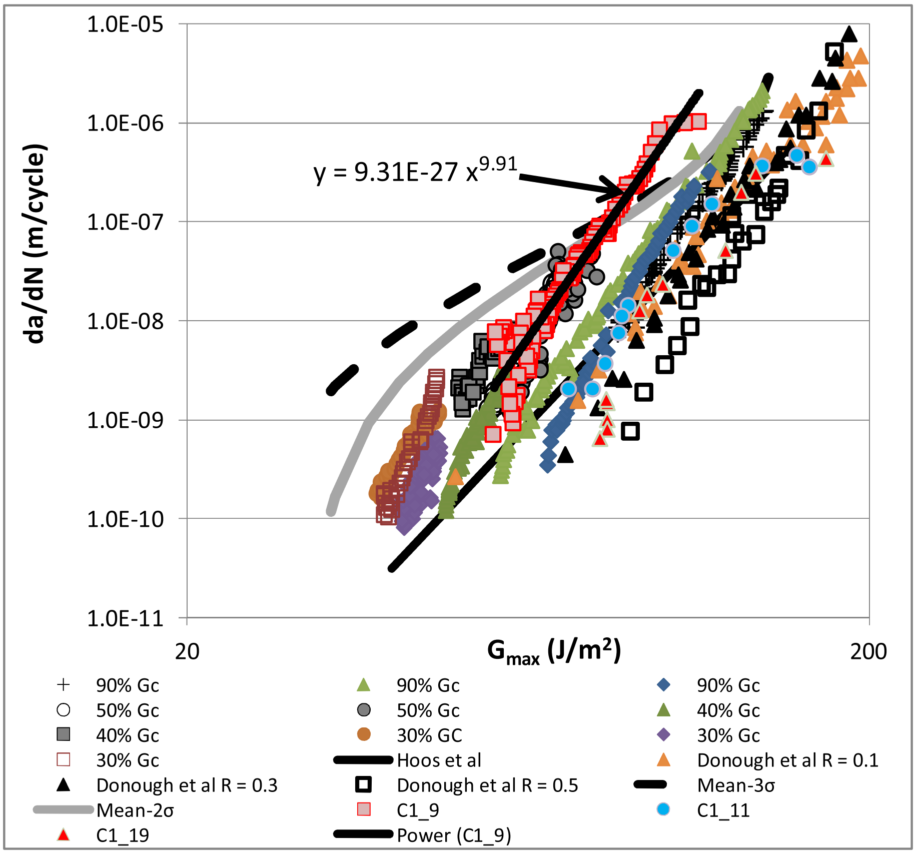

Fig. 9. Scatter in the delamination growth tests in IM7/977-3 CFRP composite laminates where the DCB tests have been started at Gmax values which represent various percentages of GC0 [42], together with test data presented in [31,39,50]. To further highlight the extent of the scatter seen in the DCB tests, the test results for specimens C1_9, C1_11 and C1_19 presented in [31] are also included. The upper-bound FCG rate curves determined using the Hartman-Schijve approach with a mean value of $\sqrt{G_{max.thr}}$ minus two standard deviations, and the mean value of $\sqrt{G_{max.thr}}$ minus three standard deviations, are also plotted. (See later for details of the calculations.)

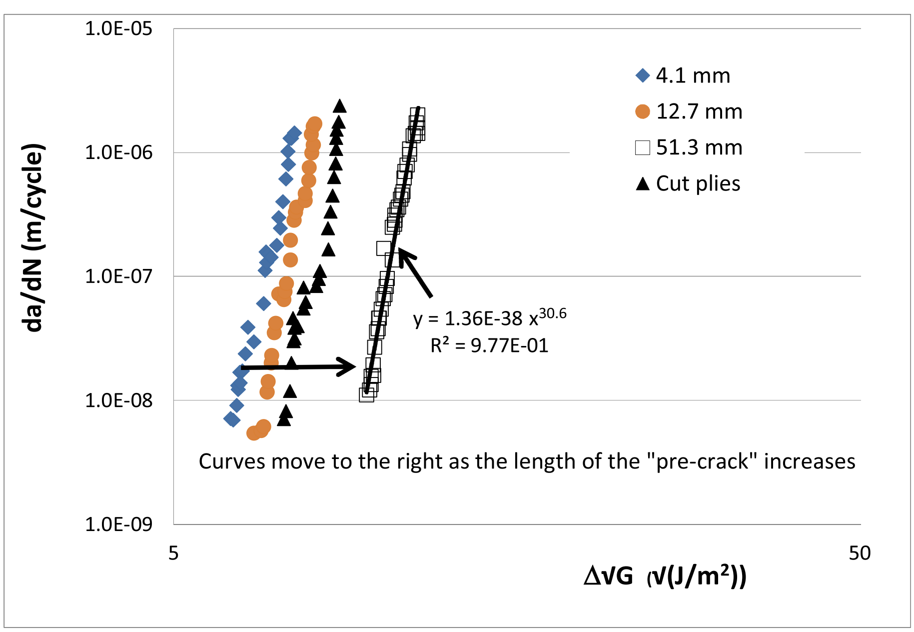

Fig. 10. Plot of da/dN versus $\Delta\sqrt{G}$ for delamination growth in DCB tests using unidirectional CFRP composite laminates for three different initial pre-crack lengths, $(a\text{-}a_0)$, as given in the figure, . adopted from [13]

which has the fastest growth rate and a means for determining this curve was proposed. This curve was termed the 'da/dN versus G_{max} at $\Delta a = 0$' curve.

3.3.4. Scatter

Murri [4,42] was the first to present experimental data that revealed the large scatter associated with da/dN versus G_{max}

curves from DCB fatigue tests. To illustrate the extent of the scatter, and hence the difficulty in determining the 'average' growth curve that is required in JSSG2006 [1], Fig. 9 presents the da/dN versus G_{max} curves presented in [4,31,39,42,50] for tests using an IM7/977-3 CFRP composite laminate. Also given are the upper-bound FCG rate curves obtained using the proposed methodology based upon a variant of the Hartman-Schijve approach, which will be discussed in more detail below.

3.3.5. Three-dimensional effects

At this point it should be noted that the da/dN versus G_{max} curves discussed above were determined using values of G that were based on the assumption that growth in the various DCB tests was two-dimensional. This assumption may not be true and the value of G may be effected by the three-dimensional shape of the delamination. This comment also applies to the various delamination studies that are discussed in Section 4.

3.3.6. Concluding remarks

The above discussions, and Figs. 9 and 10, reveal a number of interesting points:

a) The results clearly demonstrate that the level of retardation is a strong function of $(a - a_0)$, i.e. the da/dN versus $\Delta\sqrt{G}$ curve shifts to the right as the length of the starting pre-crack is increased.
b) The da/dN versus G_{max} curves given in [7,39,42] show that the delamination growth (i.e. the FCG) rate increases as the starting value of G_{max} is reduced.
c) Given the extensive scatter seen in the test data given in [4,5,31,42] and the fact that the various R-ratio tests outlined in [39] were single tests, i.e. there were no replicate tests reported, the effect of different R-ratios on the da/dN versus G_{max} curve cannot be substantiated from the results of these limited test data, since the apparent R-ratio effect may merely be due to scatter.
d) When analysing delamination growth from a fastener hole then [50] used a power-law representation associated with their da/dN versus G_{max} data, which were obtained using specimen tests that were highly retarded. They then extrapolated this relationship down to low values of da/dN. Unfortunately, this approach does not yield a conservative estimate of the da/dN versus G_{max} data, see Fig. 9; where this extrapolated power-law relationship is denoted as the 'Hoos et al.' line [50]. It also fails to account for the fact that, as seen in Section 2, the fastest growing lead delaminations in aircraft structures show little, if any, retardation. As such, when attempting to predict delamination growth from a fastener hole, the da/dN versus G_{max} curve associated with tests that have significant retardation should *not* be used, instead a 'retardation-free' upper-bound FCG curve should be employed.
e) Therefore, from the above comments, there are clearly major practical difficulties in conducting practical DCB fatigue tests which will yield reliable and valid upper-bound, and hence conservative, FCG rate curves, i.e. which are essentially 'retardation-free'.
f) Furthermore, there are no current test and analysis methods available that could prove that the results from a DCB fatigue test were sufficiently 'retardation-free' to give a reliable and valid upper-bound FCG.
g) Turning to an airworthiness perspective, the objective is to predict the lives associated with the fastest growing delamination, i.e. the lead delamination, in an aircraft structure. As we have seen such lead delaminations exhibit little retardation and may grow from relatively small naturally-occurring defects or induced damage. Therefore, in order to give reliable predictions the appropriate, and valid and conservative, DCB tests should ideally contain only relatively small initial delaminations and so be free of fibre-bridging effects. Furthermore, given the large scatter associated with DCB tests, see Fig. 9, any test program, or any subsequent analysis of the data, should allow for a significant number of replicate tests.
h) Despite the above observations recorded in the literature, the approach adopted in Section 3.2.19.1 of the United States JSSG [1] is to use an 'average growth curve'. Given the extensive scatter that can arise such an average curve is both difficult to determine and will be significantly non-conservative.

4. Representing the fatigue delamination growth and assessing the fatigue threshold

4.1. Introduction

In the previous section we noted that the FAA 'slow growth' approach to aircraft certification requires that delamination growth in polymer-matrix fibre composites under cyclic-fatigue loading be both slow and predictable. In Section 1 it was explained that until recently delamination growth in composites was usually related to the increment in the energy release-rate per cycle, ΔG, or to the maximum value of the energy-release rate, G_{max}, in a cycle. However, the logical extension of Paris' growth law to composites is to express da/dN as a function of $\sqrt{G_{max}}$ and/or $\Delta\sqrt{G}$. Indeed, as shown in [17,43] expressing da/dN as a function of ΔG often leads to the anomalous conclusion that, for a given ΔG, increasing the mean stress level reduces the delamination growth rate. This anomaly is removed if da/dN is expressed as a function of $\Delta\sqrt{G}$, see [17,43].

Other shortcomings arising from seeking to express da/dN as a function of ΔG, or G_{max}, are:

a) The dependence of both the exponent and the factor of proportionality of the power law on the mode of loading, i.e. whether Mode I, II or Mode III loading.
b) The dependence of both the exponent and the factor of proportionality on the R-ratio.
c) The dependence of both the exponent and the factor of proportionality on the scatter associated with a given FCG test.
d) The dependence of the functional relationship between da/dN and ΔG on the length of the sharpened pre-crack that is grown from the initial film-insert and the starting value of G_{max}.

These shortcomings are aptly reflected in [4,13,14,42,53] but they appear to vanish if da/dN is expressed as per the Hartman-Schijve variant of the Nasgro equation [5,7,12,17,18,27–32,34,36,38].

4.2. The Hartman-Schijve approach

As commented above, to resolve the shortcomings noted above and to accurately describe the cyclic-fatigue behaviour of polymer-matrix fibre composites, the growth rate, da/dN, should be expressed as a function of $\Delta\sqrt{G}$, which is equivalent to ΔK as used for metals. Thus, the form of the Hartman-Schijve variant of the Nasgro equation now becomes [38]:

$$da/dN = D\left[\frac{\Delta\sqrt{G} - \Delta\sqrt{G_{thr}}}{\sqrt{\{1 - \sqrt{G_{max}}/\sqrt{A}\}}}\right]^n \quad (3)$$

where D, n and A are constants and the value of A is often taken to be equivalent to the quasi-static value of the fracture energy, G_c [38], or it may be fitted [5]. The term $\Delta\sqrt{G}$ is defined by:

$$\Delta\sqrt{G} = \sqrt{G_{max}} - \sqrt{G_{min}} \tag{4}$$

The term $\Delta\sqrt{G_{thr}}$ represents the fatigue threshold and is given by:

$$\Delta\sqrt{G_{thr}} = \sqrt{G_{thr.max}} - \sqrt{G_{thr.min}} \tag{5}$$

and the subscript 'thr' in Eqs. (3) and (5) refers to the values at threshold, such that below the value of $\Delta\sqrt{G_{thr}}$ no significant FCG occurs. The value of $\Delta\sqrt{G_{thr}}$ is often best determined by ensuring that Eq. (3) fits the entire range of the data, although it may also be possible to determine its value experimentally [35,38]. Here it should be noted that the introduction of the term $\sqrt{A}$ in the denominator of Eq. (3) is somewhat similar to the use of the critical radius as suggested by Berto and Lazzarin [61]. It should also be noted that both A and $\Delta\sqrt{G_{thr}}$ are functions of the mode mix. Now, many authors [4,5,15,38,42,46–50] have chosen to express da/dN as a function of $G_{max,}$ rather than ΔG. In such cases the equivalent form of the Hartman-Schijve variant of the Nasgro equation becomes:

$$da/dN = D\left[\frac{\sqrt{G_{max}} - \sqrt{G_{max.thr}}}{\sqrt{\{1 - \sqrt{G_{max}}/\sqrt{A}\}}}\right]^n \tag{6}$$

where $\sqrt{G_{max.thr}}$ is the corresponding threshold term, which is chosen so as to ensure that Eq. (6) fits the entire range of the data.

4.3. Application of the Hartman-Schijve approach to analyse DCB fatigue test data

As a first example of the application of the variant of the Hartman-Schijve approach to DCB fatigue test data, recall that, as previously noted, [15] was the first to propose using a variant of the Hartman and Schijve equation [16] to assess delamination growth. In this test programme [15] surface-treated 2024-T3 aluminium alloy sheets, carbon/epoxy pre-preg sheets and unidirectional carbon fibre reinforced laminates, with the carbon fibres aligned along the rolling direction of the aluminium alloy, were stacked together to produce CFRP reinforced aluminium laminates (CARALL) that were subsequently fatigue tested, see [15] for more details. A designation CX was used to represent different CARALL laminates, where X is the number of carbon/epoxy pre-preg sheets in the laminate (i.e. C2, C4, C6 in the as-cured condition). The variant of the Hartman and Schijve approach was proposed since they performed DCB fatigue tests using laminates that possessed different levels of residual stresses. It was observed that the effect of the different levels of residual stress that were present in the laminates gave rise to different da/dN versus G_{max} curves, each with different values of the slopes and the fatigue thresholds. However, [17,43] subsequently revealed that when da/dN was plotted as a function of $\Delta\sqrt{G}$ the various curves were shown to coincide to give a single 'master' relationship.

It has been previously mentioned that, for a fixed R-ratio test, the apparent fatigue threshold and the da/dN versus ΔG curve both appear to be functions of the size of the initial pre-crack that is grown from the insert before the DCB fatigue test is started and of the starting value of G_{max}. Further, that the scatter associated with DCB fatigue tests can frequently be very large. (In this respect, apart from inherent material preparation factors, other factors that contribute to the scatter in the FCG rate curves have been identified, e.g. limited resolution in the measured load [32] and displacement [80]). We have also remarked on the importance of using the fatigue threshold value of the energy-release rate, instead of the quasi-static initiation value, for delamination growth in the design and assessment of composite structures subjected to fatigue loadings. In this context there are clearly major practical difficulties in conducting practical DCB fatigue tests which will yield reliable and valid upper-boundand hence conservative, FCG rate curves, i.e. which are 'retardation-free'. Furthermore, as noted above, there are no current test and analysis methods available that could prove that the results from a DCB fatigue test were sufficiently 'retardation-free' to give a reliable and valid upper-bound FCG. Therefore, following on from the successful use of the variant of the Hartman-Schijve approach to analyse the FCG in composites and in adhesively-bonded joints as shown in [5,7,12,17,27–32,34, 36,38], the questions below are very relevant. Namely, by using the variant of the Hartman-Schijve approach can we develop a methodology which allows us:

a) To calculate a lower-bound on the fatigue threshold?
b) To then calculate a corresponding valid, upper-bound curve for the fatigue growth rate of the delamination?

4.4. Application of the Hartman-Schijve approach to predict the upper-bound FCG rate curves from DCB fatigue test data

4.4.1. For an IM7/977-3 CFRP composite laminate

To address these questions, consider the various da/dN versus G_{max} curves presented in Fig. 9 for the IM7/977-3 CFRP composite laminate. The recent paper [7] has shown that the scatter in the data presented in [42], and reproduced in Fig. 9, can be represented by the Hartman-Schijve variant of the Nasgro equation, i.e. Eq. (7):

$$\frac{da}{dN} = 1.15 \times 10^{-9}\left[\frac{\sqrt{G_{max}} - \sqrt{G_{max.thr}}}{\sqrt{\{1 - \sqrt{G_{max}}/\sqrt{A}\}}}\right]^{2.24} \tag{7}$$

with A = 154 J/m^2, which corresponds to the initiation value of G_{C0} of 154 J/m^2 given in [42], and allowing for small changes in the term, $\sqrt{G_{max.thr}}$, viz: giving a mean value of 6.80 $\sqrt{(J/m^2)}$ with a standard deviation of 0.67 $\sqrt{(J/m^2)}$. To illustrate this, Fig. 11 presents a plot of da/dN versus $\left[\frac{\sqrt{G_{max}} - \sqrt{G_{max.thr}}}{\sqrt{\{1 - \sqrt{G_{max}}/\sqrt{A}\}}}\right]$ where we may see that, simply by allowing for small changes in the threshold term, $\sqrt{G_{max.thr}}$, each of the various specimen tests collapse to give a single, linear 'master' relationship.

At this point it is worthwhile noting the process used in [7] to obtain the values employed in this equation. The value of A was taken to be the fracture toughness value given in [42]. The values of da/dN were then plotted against the term $\left[\frac{\sqrt{G_{max}} - \sqrt{G_{max.thr}}}{\sqrt{\{1 - \sqrt{G_{max}}/\sqrt{A}\}}}\right]$. The values of the threshold term $G_{max.thr}$ were then adjusted until all of the various curves fell onto a single master curve. The values of D and n were then obtained by a fitting a power law to this master curve.

Therefore, let us now consider how to determine a valid, upper-bound FCG rate curve, which is 'retardation-free', and hence valid for material characterisation, material comparisons and designing and lifing studies. To achieve this it was considered that the best methodology was to use the Hartman-Schijve variant of the Nasgro equation for this composite laminate, i.e. Eq. (7), and to adopt the statistical approach suggested in [11,81], i.e. by plotting upper-bound curves obtained from using values of $\sqrt{G_{max.thr}}$ corresponding to the mean value of $\sqrt{G_{max.thr}}$ minus two standard deviations, and also the mean value minus three standard deviations. (These values are given in Table 1, and arise from the fitting of the measured FCG curves to Eq. (7) to give the single, linear, 'master' relationship, see Fig. 11, which gave a mean value of $\sqrt{G_{max.thr}}$ of 6.80 $\sqrt{(J/m^2)}$ with a standard deviation of 0.67 $\sqrt{(J/m^2)}$, as noted above). Of course, for a normal distribution the mean minus two

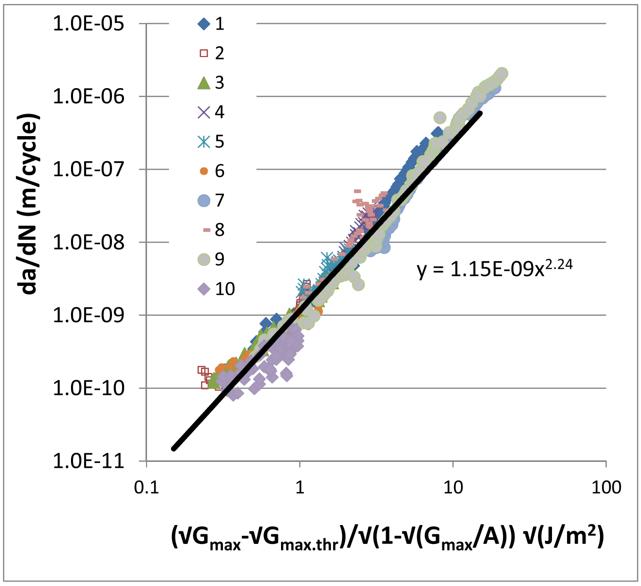

Fig. 11. Plot of da/dN versus $(\sqrt{G_{max}} - \sqrt{G_{max}}.\mathrm{thr})/\sqrt{(1-\sqrt{(G_{max}/A)})}$ for the IM7/977-3 CFRP composite laminates, . adopted from [7]

standard deviations is equivalent to a 95% confidence estimate, and a mean minus three standard deviations curve is equivalent to a 99.7% estimate.

Now, as may be seen from Fig. 9 and more clearly from Fig. 12, the upper-bound curves predicted from this proposed methodology, based upon the variant of the Hartman-Schijve approach, do indeed act as an upper-bound for virtually all the experimental data, especially at the very important part of the FCG curve where the rate of FCG is relatively very slow. (It is this part of the FCG curve that largely determines the life of a structure.) Fig. 12 also reveals that the exponent of the power law relationship between da/dN and G_{max} associated with this "upper-bound" curve is significantly reduced.

Finally, from Fig. 12, and as briefly mentioned above, it is interesting to note that at high values of da/dN the power-law representation used in [50] to represent delamination growth from a hole in a IM7/977-3 CFRP composite laminate, which in Fig. 12 is labelled the 'Hoos et al.' line, essentially coincides with the high degree of retardation FCG rate curves presented in [39]. However, at low values of da/dN this curve significantly *underestimates* the delamination growth rate curves associated with the test data presented in [42]. This is an important finding given that, as we have previously seen, the various studies on the growth of delaminations from (a) a hole being mis-drilled, (b) a ply drop-off, and (c) impact damage revealed that the fastest growing delaminations showed little apparent retardation. These comments therefore reinforce the need to base service-life estimates on da/dN curves that are associated with an upper-bound FCG rate curve which shows no, or only very little, retardation.

4.4.2. For an IM7/8552 CFRP composite laminate

To further illustrate this proposed methodology, based upon using the variant of the Hartman-Schijve approach to predict an upper-bound FCG rate curve, let us consider the data presented in [4] for delamination growth in DCB fatigue tests using an IM7/8552 CFRP composite laminate. The recent paper [7] has also shown that the scatter in the data presented in [4], and reproduced in Fig. 13, can be represented by the Hartman-Schijve variant of the Nasgro equation to give a single, linear 'master' relationship, as shown in Fig. 14, i.e. Eq. (8):

$$\frac{da}{dN} = 1.6 \times 10^{-9}\left[\frac{\sqrt{G_{max}} - \sqrt{G_{max.thr}}}{\sqrt{\{1-\sqrt{G_{max}}/\sqrt{A}\}}}\right]^{2.65} \tag{8}$$

with A = 240 J/m^2, as given in [4] and in Table 2, and simply allowing for small changes in the term $\sqrt{G_{max.thr}}$, viz: giving a mean value of 8.08 $\sqrt{(J/m^2)}$ with a standard deviation of 0.60 $\sqrt{(J/m^2)}$. Again, Eq. (8) yields a single, linear 'master' relationship, as shown in Fig. 14, for all the various curves given in Fig. 13.

As before, our methodology is to calculate upper-bound FCG curves for the results shown in Fig. 13 using the variant of the Hartman-Schijve approach, as embodied in Eq. (8). To this end, Fig. 13 contains plots obtained using the Hartman-Schijve variant of the Nasgro equation for this composite laminate, i.e. Eq. (8), with the values of $\sqrt{G_{max.thr}}$ corresponding to the mean value of $\sqrt{G_{max.thr}}$ minus two standard deviations, and the mean value minus three two standard deviations. (These values are given in Table 2, and again arise from the fitting of the measured FCG curves, see Fig. 13, to Eq. (8) to give the single, linear 'master' relationship, see Fig. 14. This gave a mean value of $\sqrt{G_{max.thr}}$ to be 8.08 $\sqrt{(J/m^2)}$ with a standard deviation of 0.60 $\sqrt{(J/m^2)}$ [7], as noted above.) When the values of $\sqrt{G_{max.thr}}$ corresponding to the mean value minus two, or three, standard deviations are used to calculate the upper-bound for the FCG rate curve, then two upper-bound curves are obtained which encompass all the experimental data points, see Fig. 13. Fig. 13 also reveals that the exponent of the power law relationship between da/dN and G_{max} associated with this "upper-bound" curve is significantly reduced.

4.4.3. Scatter in the ESIS Mode I round-robin tests

Refs. [5,32] presented the results of a European Structural Integrity Society (ESIS) Panel TC4 round-robin study of the Mode I DCB fatigue delamination in G30-500/R5276 composite laminates, with R = 0.1. The results of these tests are shown in Fig. 15.

The recent paper [5] has again shown that these data can also be represented by the Hartman-Schijve variant of the Nasgro equation, i.e. Eq. (9), as shown in Figs. 15 and 16. The relevant equation is:

$$\frac{da}{dN} = 4.0 \times 10^{-10}\left[\frac{\sqrt{G_{max}} - \sqrt{G_{max.thr}}}{\sqrt{\{1-\sqrt{G_{max}}/\sqrt{A}\}}}\right]^{2.3} \tag{9}$$

The values of A and $\sqrt{G_{max.thr}}$ needed to obtain this single, linear 'master' relationship, as represented by Eq. (9) and shown in Fig. 16, are given in Table 3. In these tests there were significant differences in the values of the apparent quasi-static toughness, A, seen in the various tests from the different laboratories. This observation is agreement with the observations from Fig. 15, where the onset on rapid growth is seen to vary significantly in the various tests. This observation is consistent with the effects of fibre-bridging retarding the crack growth rate in the DCB fatigue tests.

Now, Eq. (9), together with the values of A and $\sqrt{G_{max.thr}}$ given in Table 3, were first used to predict the full experimental FCG curves of da/dN versus the value of G_{max} shown in Fig. 15 for the DCB fatigue tests that have been conducted by the different laboratories. The predicted relationships are shown in Fig. 15 and, as maybe seen, there is good agreement between the experimental data

Table 1
Values of $\sqrt{G_{max.thr}}$ and A used to predict the upper-bound FCG rate curves shown in Figs. 9 and 12.

	$\sqrt{G_{max.thr}}$ ($\sqrt{(J/m^2)}$)	A (J/m^2)
Mean – 2 σ	5.46	154
Mean – 3 σ	4.79	154

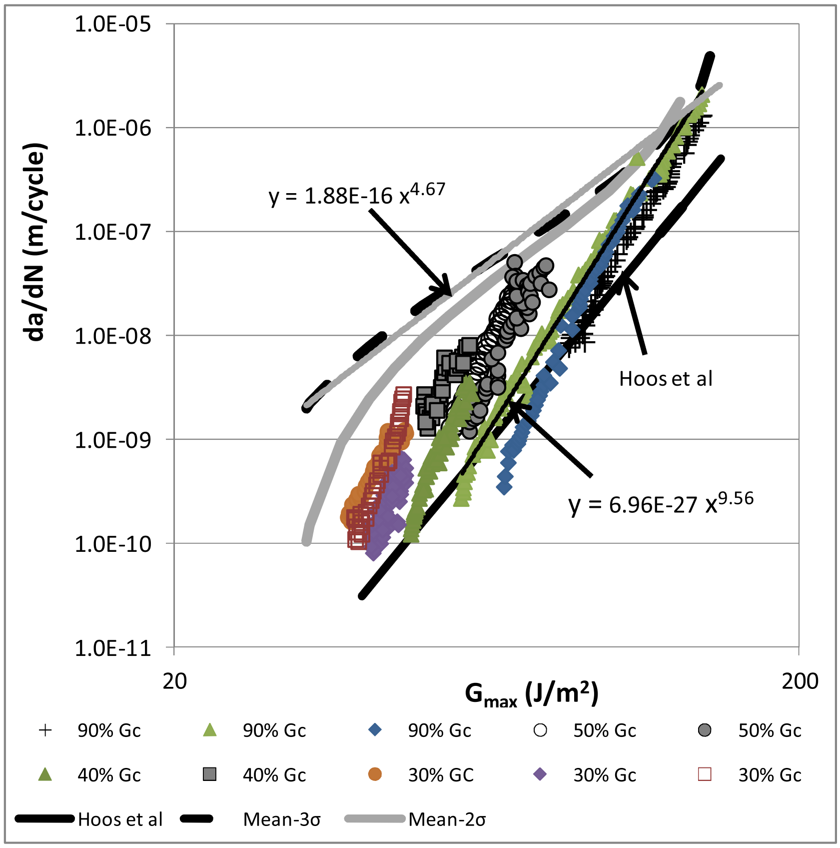

Fig. 12. Comparison of the delamination growth curves given in [42,50] for the IM7/977-3 CFRP composite laminates. The upper-bound FCG rate curves are also plotted and were determined using the Hartman-Schijve approach with a mean value of $\sqrt{G_{max.thr}}$ minus two standard deviations, and the mean value minus three standard deviations. Lines of best fit to the Mean -3σ and the 90% G_C data sets are also shown.

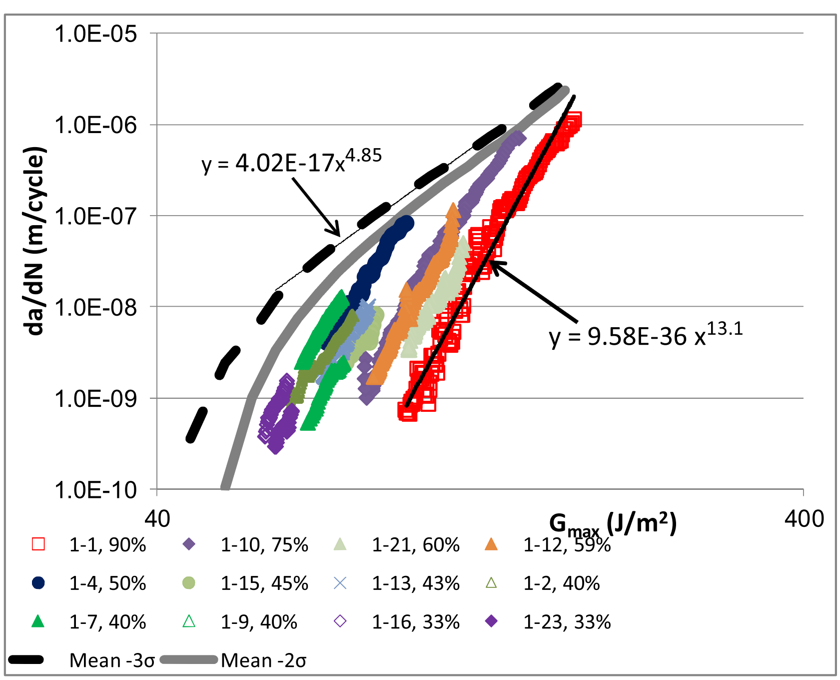

Fig. 13. Scatter in the delamination growth tests in the IM7/8552 CFRP composite laminate at various percentages of G_{C0} [4] together with the predicted upper-bound FCG rate curves that were determined using the Hartman-Schijve approach with a mean value of $\sqrt{G_{max.thr}}$ minus two standard deviations, and the mean value minus three standard deviations.

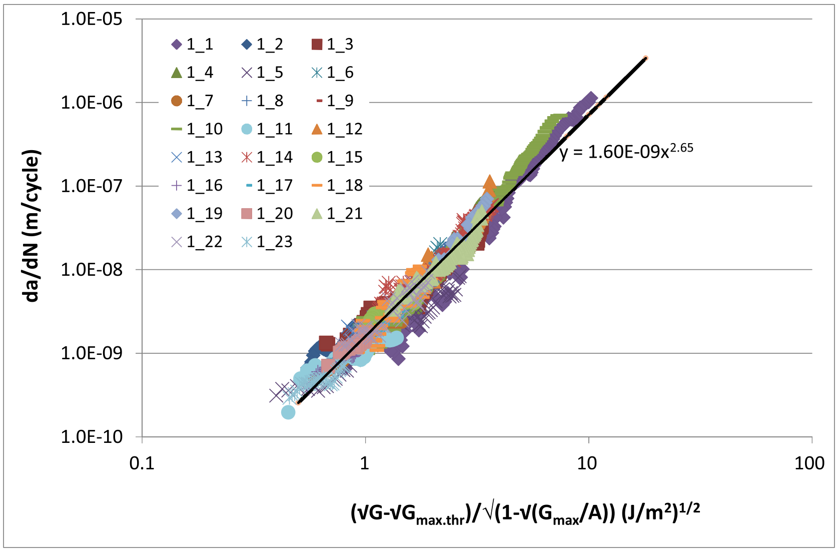

Fig. 14. Plot of da/dN versus $(\sqrt{G_{max}} - \sqrt{G_{max.thr}})/\sqrt{(1 - \sqrt{(G_{max}/A)})}$ for the IM7/8552 CFRP composite laminates, . adopted from [7]

Table 2
Values of $\sqrt{G_{max.thr}}$ and A used to predict the upper-bound FCG rate curves shown in Fig. 13.

	$\sqrt{G_{max.thr}}$ $(\sqrt{(J/m^2)})$	A (J/m^2)
Mean – 2 σ	6.88	240
Mean – 3 σ	6.28	240

and the predicted curves computed using the variant of the Hartman and Schijve approach. Thus, the cyclic-fatigue behaviour for the DCB fatigue tests under Mode I loading of the composite laminate may indeed be very well represented using this form of the Hartman and Schijve equation.

Secondly, turning to the prediction of the upper-bound FCG curves, then Fig. 15 also contains plots of the predicted upper-bound FCG curves obtained from using our methodology of employing the Hartman-Schijve variant of the Nasgro equation for this composite laminate. Namely, we used Eq. (9), with the values of $\sqrt{G_{max.thr}}$ corresponding to the mean value of $\sqrt{G_{max.thr}}$ minus two standard deviations, and the mean value minus three standard deviations, see Table 4. (These values again arise from the fitting of the measured FCG curves, see Fig. 15, to Eq. (9) to give the single, linear 'master' relationship, see Fig. 16. The value used for A in our methodology was taken to be the lowest value determined, since this value represents the test condition when no significant retardation is present.) This process again yields very good upper-bound estimates for the FCG rate data, where all the experimental data are encompassed by these upper-bound curves. Fig. 15 again reveals that the exponent of the power law relationship between da/dN and G_{max} associated with this "upper-bound" curve is significantly reduced.

4.4.4. Concluding remarks

These results shown above illustrate how it is possible to use the Hartman-Schijve variant of the Nasgro equation, i.e. Eqs. (3) or (6) depending upon the choice of the energy-release rate term, to create an approximate upper-bound da/dN versus G_{max}, or ΔG, curve. This upper-bound curve represents the worst-case for the FCG rate since no, or very little, retardation of the growth of the delamination is present. This curve may therefore now be used for accurately assessing (a) the characterisation and comparison of different composite materials, and (b) the design and lifing of aircraft structures where material allowable properties have to be inputted into a delamination growth analysis.

One aspect of this proposed methodology is based on the idea of taking the 'mean minus two, or three, standard deviations' to employ in the calculations. This idea finds support from the work of Rouchon [11]. He essentially commented that, if sufficient data points exist to give an accurate assessment of the true mean value of $\sqrt{G_{max.thr}}$, the mean value of $\sqrt{G_{max.thr}}$ minus two standard deviations may be used. If not, then the mean value of $\sqrt{G_{max.thr}}$ minus three standard deviations should be used. Further, this approach finds additional support from the statements by Niu [81]. He has reviewed the statistical procedures used to derive material allowable properties for composite materials, to input into design and lifing analyses, under the basic headings of 'A' and 'B':

a) 'A' basis: The mechanical property value indicated is the value above which at least 99% of the population of values is expected to fall with the confidence of 95%. This value is used to design and lifing a single member where the loading is such that its failure would result in a loss of structural integrity.
b) 'B' basis: The mechanical property value indicated is the value above which at least 90% of the population of values is expected to fall with the confidence of 95%. This value is used in the design and lifing of redundant or fail-safe structural analyses, where the loads may be safely distributed to other members.

Obviously the 'A' basis is essentially equivalent to the idea of taking the mean value minus three standard deviations, and the 'B' basis to the idea of the mean value minus two standard deviations.

Let us next briefly address the issue of design. As previously noted a 'no growth' design requires an accurate knowledge of the fatigue threshold. However, as we have seen in Figs. 12, 13 and 15 the scatter in the fatigue threshold can be very large. The Hartman-Schijve variant has the potential to yield a (more) conservative estimate of the fatigue threshold.

Finally, it should be noted that, as may be seen from Figs. 12, 13 and 15, for the experimentally-measured FCG rate curves the values of the exponent in the power law for the relationship between da/dN and G_{max}, for the linear regions of these curves, may be rel-

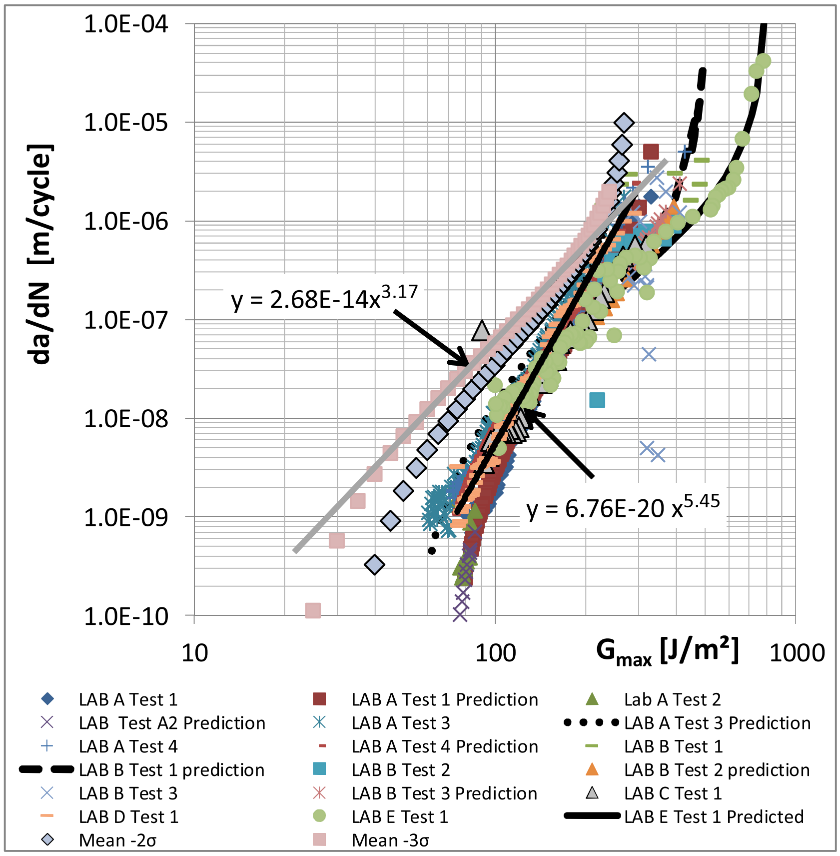

Fig. 15. Scatter in the delamination growth tests in the G30-500/R5276 CFRP composite laminates where the DCB fatigue tests have been conducted by different laboratories, from [5,32]. The predicted FCG curves for the results for the different laboratories computed using the Hartman-Schijve approach are shown. The upper-bound FCG curves are also plotted and were determined using the Hartman-Schijve approach with the mean value of $\sqrt{G_{max.thr}}$ minus two standard deviations, and the mean value minus three standard deviations.

atively large, i.e. the measured FCG curves are relatively very steep. This has also been previously observed [26,41,82]. This led [41] to conclude that:

> *'For composites, the exponents for relating propagation rate to energy release-rate have been shown to be high especially in Mode I. With large exponents, small uncertainties in the applied loads will lead to large uncertainties (of at least one order of magnitude) in the predicted delamination growth rate. This makes the derived power-law relationships unsuitable for design purposes.'*

However, in contrast, this exponent for the upper-bound FCG rate curve, predicted using our methodology based upon using a variant of the Harman-Schijve approach, is typically significantly lower in value than for the experimentally-measured FCG curves, see Figs. 12, 13 and 15. This suggests that our methodology of using the predicted, 'retardation-free', upper-bound FCG rate curve may enable engineers to indeed allow for some fatigue crack growth to be permitted when designing and lifing composite structures. This aspect is currently being studied and it is intended that it will form the basis for a paper in the future.

5. Conclusions

The growth of delaminations in polymer-matrix fibre composites under cyclic-fatigue loading in operational aircraft structures has always been a very important factor which has the potential to significantly affect the service-life of such structures. The recent introduction by the FAA of a 'slow growth' approach to the certification of composites has focused attention on the experimental data and the analytical tools needed to assess the growth of delaminations under fatigue loads. Therefore, a main emphasis of the present paper has been to address the topic of the growth of delaminations in polymer-matrix fibre composites under cyclic-fatigue loading using a fracture-mechanics approach.

This paper has shown that experimental data suggest that the fastest growing, i.e. lead, delaminations that arise under cyclic-fatigue loading of real structures or components from mis-drilled holes, ply drop-offs and impact damage show no, or only little, retardation. Such retardation typically arises due to fibre-bridging developing across the faces of the delamination as the fatigue crack advances. Therefore, of course, the FCG data that is ascertained in the laboratory, and then employed as a material allowable property to design and life the structure, as well as for the characterisation and comparison of composite materials, must also exhibit no, or only minimal, retardation. It is also noteworthy that fibre-bridging effects, that arise in typical laboratory fatigue tests, can give rise to the large scatter that is typically seen in fatigue tests on composite materials.

Thus, for all the above reasons, it is suggested that the typical DCB fatigue tests performed in order to determine a delamination growth curve should focus on ensuring that the fastest possible growth curve is measured, rather than any FCG rate curve that is associated with retardation effects, possibly arising from extensive fibre-bridging. Notwithstanding, the experimental data reveals that retardation effects cannot usually be avoided. This is due to the major experimental difficulties that are encountered when trying to undertake 'retardation-free' DCB fatigue tests, i.e. with no, or only minimal, fibre-bridging present across the faces of the

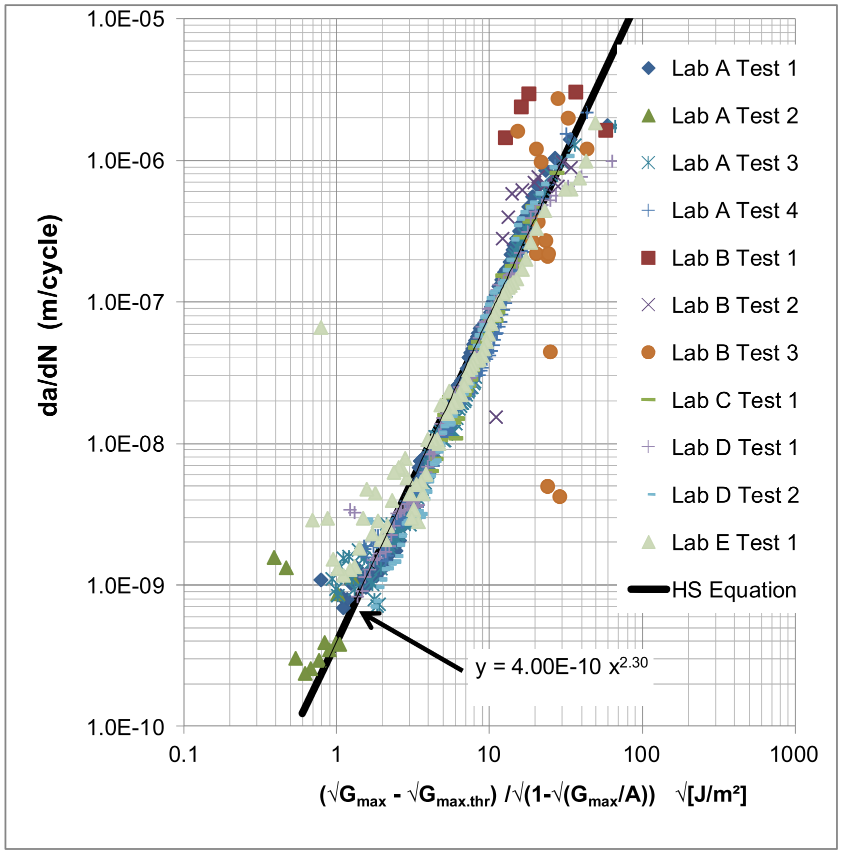

Fig. 16. Plot of da/dN $(\sqrt{G_{max}} - \sqrt{G_{max.thr}})/\sqrt{(1-\sqrt{(G_{max}/A)})}$ for the test data given in [5,32] for the G30-500/R5276 CFRP composite laminates.

Table 3
The values of A and $\sqrt{G_{max.thr}}$ needed to obtain the 'master' relationship shown in Fig. 16 for the individual fatigue curves of da/dN versus G_{max} shown in Fig. 15.

Specimen	A (J/m^2) from [5]	$\sqrt{G_{max.thr}}$ ($\sqrt{(J/m^2)}$)	Specimen	A (J/m^2), from [5]	$\sqrt{G_{max.thr}}$ ($\sqrt{(J/m^2)}$)
A.1	350	8.4	C.1	350	8.1
A.2	350	8.4	C.2	350	8.1
A.3	280	7.1	C.3	280	7.1
A.4	320	7.4	C.4	280	5.5
B.1	700	4.5	C.5	320	7.4
B.2	280	7.7	D.1	500	7.7
B.3	320	7.4	D.2	500	8.4
E.1	350	8.1			
E.2	350	8.4			

Table 4
Values of $\sqrt{G_{max.thr}}$ and A used to predict the upper-bound FCG rate curves shown in Fig. 15.

	$\sqrt{G_{max.thr}}$ ($\sqrt{(J/m^2)}$)	A (J/m^2)
Mean – 2σ	5.61	280
Mean – 3σ	4.52	280

delamination. The experimental data also reveals that DCB fatigue test results usually show a great deal of scatter, which may arise from fibre-bridging developing during the test. It is therefore very difficult to determine a meaningful 'average' delamination growth curve. The same comments are true with respect to determining a valid value of the fatigue threshold, below which no significant FCG occurs.

Thus, a methodology is needed for estimating a valid upper-bound curve which encompasses all the experimental data and provides a conservative FCG curve and which is representative of a composite laminate exhibiting no, or only very little, retardation under fatigue loading. Such a valid, upper-bound curve can then employed for (a) the characterisation and comparison of composite materials, (b) a 'no growth' design, (c) for assessing if a delamination, that is found in an in-service aircraft, will grow and (d) the design and lifing of in-service composite aircraft structures where material allowable properties have to be inputted into a delamination growth analysis.

A novel methodology based on using a variant of the Hartman-Schijve approach has been proposed to access this valid, upper-bound FCG rate curve, which may be thought of as a material allowable property. This methodology uses two options: either taking the mean value of the threshold, $\sqrt{G_{max.thr}}$, minus two standard deviations, or the mean value minus three standard deviations. (The mean value of $\sqrt{G_{max.thr}}$ and its standard deviation are calculated from analysing the measured DCB fatigue test data using the variant of the Hartman-Schijve approach.) The former option of taking the mean value of the threshold, $\sqrt{G_{max.thr}}$, minus two standard deviations gives the less conservative, of the two, upper-bound fatigue growth curves predicted via the Hartman-Schijve variant. If, as in [11,81], a more conservative curve is required, for say primary aircraft structure, an approach based on the mean value of $\sqrt{G_{max.thr}}$ minus three standard deviations may be preferred.

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